

Topics : Relative Motion, Work, Power and Energy

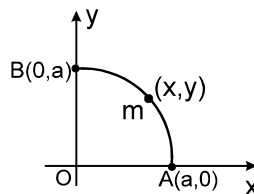
Type of Questions

		M.M., Min.
Single choice Objective ('-1' negative marking) Q.1	(3 marks, 3 min.)	[3, 3]
Subjective Questions ('-1' negative marking) Q.2 to Q.3	(4 marks, 5 min.)	[8, 10]
Comprehension ('-1' negative marking) Q.4 to Q.6	(3 marks, 3 min.)	[9, 9]
Match the Following (no negative marking) (2 × 4) Q.7	(8 marks, 10 min.)	[8, 10]

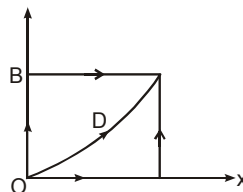
1. Two objects moving along the same straight line are leaving point A with an acceleration a , $2a$ & velocity $2u$, u respectively at time $t = 0$. The distance moved by the object with respect to point A when one object overtakes the other is :

- (A) $\frac{6u^2}{a}$ (B) $\frac{2u^2}{a}$ (C) $\frac{4u^2}{a}$ (D) none of these

2. A particle of mass 'm' moves along the quarter section of the circular path whose centre is at the origin. The radius of the circular path is 'a'. A force $\vec{F} = y\hat{i} - x\hat{j}$ newton acts on the particle, where x , y denote the coordinates of position of the particle. Calculate the work done by this force in taking the particle from point A (a, 0) to point B (0, a) along the circular path.

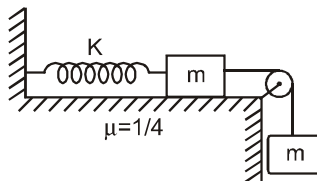


3. A particle is moved along the different paths OAC, OBC & ODC as shown in the fig. Path ODC is a parabola, $y = 4x^2$. Find the work done by a force $\vec{F} = xy\hat{i} + x^2y\hat{j}$ on the particle along these paths. Is this force a conservative force ?

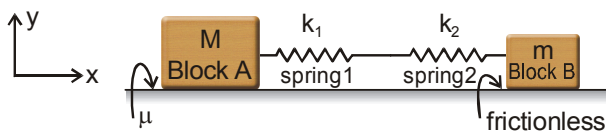


COMPREHENSION

Consider the system shown below, with two equal masses m and a spring with spring constant K . The coefficient of friction between the left mass and horizontal table is $\mu = 1/4$, and the pulley is frictionless. The string connecting both the blocks is massless and inelastic. The system is held with the spring at its unstretched length and then released.



4. The extension in spring when the masses come to momentary rest for the first time is
 (A) $\frac{3mg}{2K}$ (B) $\frac{mg}{2K}$ (C) $\frac{mg}{K}$ (D) $\frac{2mg}{K}$
5. The minimum value of μ for which the system remains at rest once it has stopped for the first time is
 (A) $\frac{1}{\sqrt{3}}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{1}{\sqrt{2}}$
6. If the string connecting both the masses is cut just at the instant both masses came to momentary rest for the first time in question 5, then maximum compression of spring during resulting motion is (Take $\mu = 1/4$)
 (A) $\frac{2mg}{3K}$ (B) $\frac{mg}{2K}$ (C) $\frac{mg}{K}$ (D) $\frac{1mg}{3K}$
7. Two blocks A and B of masses m and M are placed on a horizontal surface, both being interconnected with a horizontal series combination of two massless springs 1 and 2, of force constants k_1 and k_2 respectively as shown. Friction coefficient between block A and the surface is μ and the springs are initially non-deformed. Now the block B is displaced slowly to the right by a distance x , and it is observed that block A does not slip on the surface. Block B is kept in equilibrium by applying an external force at that position. Match the required information in the left column with the options given in the right column.



Left column

- (A) Friction force on block A by the surface
- (B) Force by spring 1 on block A
- (C) Force exerted by spring 2 on spring 1.
- (D) External force on block B.

Right column

- (p) $k_1 x (-\hat{i})$
- (q) $\mu Mg (-\hat{i})$
- (r) $\frac{k_1 k_2 x}{k_1 + k_2} (\hat{i})$
- (s) $\frac{k_1 k_2 x}{k_1 + k_2} (-\hat{i})$

Answers Key

DPP NO. - 39

1. (A) 2. $-\frac{\pi a^2}{2}$
3. $w_{OAC} = 8 \text{ J}$, $w_{OBC} = 2 \text{ J}$; $w_{ODC} = 19/3 \text{ J}$, No
4. (A) 5. (B) 6. (C)
7. (A) s (B) r (C) r (D) r

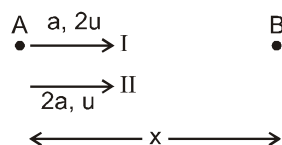
Hint & Solutions

DPP NO. - 39

1. Let both will meet at point B

$$x = 2ut + \frac{1}{2} at^2$$

$$x = ut + \frac{1}{2} (2a)t^2$$



$$\text{So } 2ut + \frac{1}{2} at^2 = ut + at^2$$

$$ut = \frac{1}{2} at^2 \Rightarrow t = \frac{2u}{a}$$

$$\text{So } x = 2u\left(\frac{2u}{a}\right) + \frac{1}{2}a\left(\frac{2u}{a}\right)^2 = \frac{6u^2}{a}$$

2. Work done by force F ;

$$w = \int \mathbf{F} \cdot d\mathbf{r} = \int (y\hat{i} - x\hat{j}) \cdot (dx\hat{i} + dy\hat{j})$$

$$= \int (ydx - xdy) \dots\dots\dots(1)$$

$$\because x^2 + y^2 = a^2 \quad \therefore xdx + y dy = 0$$

$$\Rightarrow W = \int \left(y \left(\frac{-ydy}{x} \right) - xdy \right)$$

$$= - \int \frac{(x^2 + y^2)}{x} dy$$

$$= - \int_0^a \frac{a^2}{\sqrt{a^2 - y^2}} dy = - \frac{\pi a^2}{2}$$

Alternate Method

It can be observed that the force is tangent to the curve at each point and the magnitude is constant. The direction of force is opposite to the direction of motion of the particle.

\therefore work done = (force) \times (distance)

$$= -\sqrt{x^2 + y^2} \frac{\pi a}{2} = -a \times \frac{\pi a}{2} = -\frac{\pi a^2}{2} \text{ J}$$

Ans. $w = -\frac{\pi a^2}{2} \text{ J}$

3. $(W_F)_{OAC} = \int (xy dx + x^2 y dy)$

$$= \int_0^A (xy dx + x^2 y dy) + \int_A^C (xy dx + x^2 y dy)$$

ON OA path ;

$y = 0$, $dy = 0$ and on AC path

$x = 1$, $dx = 0$

$$(W_F)_{OAC} = \int_0^A (0 \cdot dx + 0 \cdot dy) + \int_{y=0}^{y=4} (0 + 1y dy) = 8 \text{ J}$$

$$(W_F)_{OBC} = 0 + \int_B^C (xy dx + x^2 y dy)$$

$$= \int_{x=0}^1 \{x \cdot 4 dx + x^2 \cdot 4(0)\} = 2 \text{ J}$$

$$(W_F)_{ODC} = \int_{y=4x^2} (xy dx + x^2 y dy)$$

$$= \int_0^1 (x \cdot 4x^2 dx + x^2 \cdot 4x^2 \cdot 8x dx) = 1 + \frac{32}{6}$$

$$= \frac{19}{3} \text{ J}$$

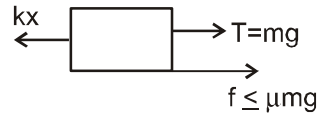
4. From work energy theorem, the masses stop when total work done on them is zero.

$$W = mgx - \frac{1}{2} kx^2 - \mu mgx = 0$$

$$\therefore \frac{2mg}{k} (1 - \mu) = \frac{3mg}{2k}$$



5. When the masses are stopped at this value of x , the forces on left mass for it to remain at rest is zero



$$\Rightarrow kx = mg + f$$

$$\Rightarrow k \frac{2mg}{k} (1 - \mu) \leq mg + \mu mg$$

$$\therefore \mu \geq 1/3 \quad \therefore \text{least value of } \mu \text{ is } 1/3.$$

6. At the instant string is cut, let the extension in spring be x_0 . The maximum compression x will occur for spring when left block comes to rest first time after the string is cut

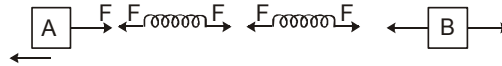
$$\therefore \text{From work energy Theorem } \Delta W = 0$$

$$\frac{1}{2} kx_0^2 - \frac{1}{2} kx^2 - \mu mg (x + x_0) = 0$$

$$x_0 = \frac{3mg}{2k} \quad \text{and} \quad \mu = \frac{1}{4}$$

$$\text{solving we get } x = \frac{mg}{k}$$

7. The free body diagram (FBD) is :



Tension in both springs will be same

(they are massless)

$$F = K_2 X_2 = K_1 X_1 \quad \text{and} \quad X_1 + X_2 = X$$

$$\therefore X_1 = \frac{K_2}{K_1 + K_2} X, \quad X_2 = \frac{K_1}{K_1 + K_2} X$$

$$\therefore F = \frac{K_1 K_2}{K_1 + K_2} X$$

$$f = F = \frac{K_1 K_2}{K_1 + K_2} X(-\hat{i})$$

$$\therefore a \rightarrow S, b \rightarrow R, c \rightarrow R, d \rightarrow R$$

